Chapter 1

Uncertainty in Dynamically Changing Input Data

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The main objective of multiple criteria decision making models is to select an alternative, from a finite number, regarding a set of pre-defined criteria. Usually, this type of problems includes two main tasks, rating the alternatives regarding each criterion and then ranking them. Once a decision is made (alternative selected) the problem is solved. However, for situations involving reaching consensus or requiring several steps before reaching a final decision, we must consider a dynamic and adaptable decision model, which considers previous solutions.

In this work we introduce multiple criteria dynamic decision making models (MCDDM) and discuss contributions to deal with the difficult problem of imprecision in dynamically changing input data. To illustrate the approach, a simplified example of autonomous spacecraft landing is presented.

1.1. Problem Statement

In general, the aim of Multiple Criteria or Multiple Attribute Decision Making problems [1] (henceforth called multiple criteria) is to find the best compromise solution from all feasible alternatives assessed by pre-defined criteria (or attributes). This type of problems is widespread in real life situations[1,2] . A Multiple Criteria Decision Making (MCDM) problem is usually modeled as a decision matrix, as depicted in Eq. (1.1). Basically, a MCDM model includes two phases[3] : (a) classify the alternatives regarding each criterion and then aggregate the respective classifications to obtain ratings per alternative; (b) order the alternatives (ranking), where the highest rate usually corresponds to the "best" alternative to be selected.

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$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_m \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} & \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \end{array} \tag{1.1}$$

where A_i is the alternative $i(= 1,...,n)$, C_j is the attribute $j(= 1,...,m)$ and x_{ij} is the evaluation (or the satisfaction) of alternative A_i regarding attribute C_j.

In this work the context is Multiple Criteria Dynamic Decision Making (MCDDM) models, specifically decision paradigms with several iterations, which need to take into account past/historic information (feedback) to reach a decision (or decision consensus). In general, dynamic systems usually have spatial-temporal limitations such as, the search space varies for each step (iteration) and there are several steps (iterations) to reach a final decision [4] . According to Richardson and Pugh [5] system dynamics problems have two main features: “they involve quantities which change over time”; and “involves the notion of feedback”. Moreover, when we are dealing with dynamic processes, the evaluation/selection process is more sophisticated because it must include information from previous iterations (historic data). Henceforth, this process is called “dynamic evaluation process” because it involves aggregation of past and current ratings, as well as selection of a sub-set of possible “good” candidates for the next iteration (historical feedback).

One important requirement that both static MCDM methods [1] and MCDDM methods face is that data must be both numerical and principally comparable. When in presence of both qualitative and quantitative variables, we need to transform and normalize the data to compare them.

In MCDDM a big challenge is how to represent dynamic input changing data. Here we also address another important aspect of many multicriteria decision problems: uncertainty in input data. Sometimes, there is a lack of confidence and accuracy on the available input data and we must devise ways of incorporating this uncertainty on the transformed input data. All these aspects fall under a general umbrella of data preparation [6] and they constitute the basis for dynamic spatial-temporal decision models.

Our main objective is to present contributions for handling the data preparation process in MCDDM models with imprecise input data. We will address four main aspects of any dynamic spatial-temporal decision problem:

(I) How to represent and normalize the input parameters within a dynamically changing input data;
(II) How to deal with uncertainty in dynamically changing input data;
(III) How to consider different criteria importance, depending on criteria satisfaction and decision process phase (iteration);
(IV) How to use feedback past information from the dynamic model to evaluate the alternatives at each decision point.

The first three questions belong to known literature processes about data preparation and multicriteria processes [1,6,7] , respectively, and they will be discussed in detail. The fourth aspect is mostly concerned with suitable operators to aggregate current and past information that should be considered at each iteration of the decision process. The latter will only be addressed here briefly because the authors already proposed some contributions for this process [8] .

Summarizing, the aim of single or dynamic Multiple Criteria Decision Making problems is to find the best compromise solution from all feasible alternatives, assessed by pre-defined criteria (or attributes) [3,7] . There are many methods and techniques to deal with static MCDM [1,3,7] . However, when facing dynamic multiple criteria decisions, where several iterations are at stake and feedback from step to step is required, there are few contributions in the literature (see for example 4,9–11).

To better explain our proposed contributions to deal with uncertainty in dynamically changing input data during the temporal decision process, we use a simplified example of a site selection problem for spacecraft landing on planets [12] . This case study requires a dynamic and adaptable decision model to ensure robust and smooth site selection along the final descent.

1.2. Description of Methodology

In this chapter we present an architecture for a multiple criteria dynamic decision model. This architecture involves two important aspects: how to perform a correct data preparation when we are dealing with dynamically changing input data and how to deal with a complex evaluation process, which includes historic information to ensure a truly dynamic and adaptable decision algorithm. In Figure 1.1 we depict the conceptual design for the dynamic decision model.

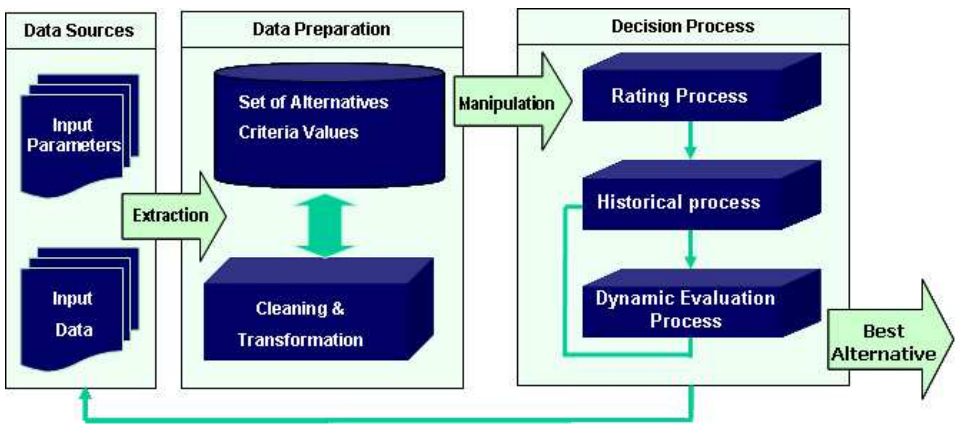

Fig. 1.1. Conceptual design of the dynamic decision model.

The two main phases, shown in Figure 1.1, are:

(A) Phase A. Data preparation in dynamic environments- This process deals with transformation of inputs for the decision model. Its main tasks are to identify the criteria and the alternatives of the problem and then to clean and transform the input data to obtain parameters/variables that can be manipulated to achieve a decision. For this phase we propose six steps that are detailed in Sec. 1.2.1.
(B) Phase B. Decision Evaluation Process with Feedback - This process starts with rating the current alternatives (assessed by criteria) and then performs evaluation of alternatives, considering rating value plus the historic information available. Finally, the historic sub-set is created and is provided as feedback for the next iteration. This process goes on until a stoping criterion is reached.

As mentioned, in this work we focus on the first step, hence more emphasis is put on Phase A in the remaining sections.

1.2.1. *Phase A. Data Preparation in Dynamic Environments*

Here we discuss our proposed data preparation steps for handling dynamically changing input data, in presence of uncertainty. As mentioned, one important requirement of MCDM methods ,[1] both static and dynamic, is that data must be both numerical and principally comparable. When in presence of both qualitative and quantitative variables, we need to transform and normalize the data to compare them. This work also addresses an important aspect of many multiple criteria decision problems: uncertainty in input data. When dealing with MCDDM problems there are three important questions to answer: how to define and represent the variables? Is the raw input data extracted accurate and do we have confidence in it? Do we know if the search space includes all possible alternatives to be evaluated at each step? Our proposed data preparation steps for MCDDM with uncertainty are:

- Step 1. Parameters identification - This process is characterized by identifying the input variables (problem understanding). In a multiple criteria decision making environment these correspond to determine criteria for assessing candidate alternatives, as well as identifying the set of candidate alternatives.[3]
- Step 2. Transformation - Data is transformed from a raw state into data suited for decision support, by normalizing and representing the input variables with fuzzy membership functions. This process is usually called fuzzification [2] and involves choosing the topology for the fuzzy sets representing the input variables. With a fuzzy logic approach we can guarantee normalized and comparable data definitions, besides allowing us to deal, in a simple fashion, with imprecision in data.
- Step 3. Cleaning - After defining the fuzzy set functions, for each criterion (i.e., representation and normalization) all candidate alternatives with membership values of zero, in any criterion, are discarded. Moreover, any other constraints that

alternatives are required to obey will be used to further clean the data, thus further reducing the set of candidate alternatives. Another aspect of cleaning is how to handle missing input data (e.g., missing records, gaps in collect data etc.). A possible solution could be to use the previous value when there is missing data[13] .

- Step 4. Fusion - transformed and cleaned data from each criterion is integrated using a conjunctive method [7] to define the general search space, i.e., number of alternatives to be evaluated with the defined criteria, per iteration. The Conjunctive method is an outranking method[7] so it does not give a score for each alternative like most common scoring methods, such as MaxiMin, WSM, WPM and TOPSIS. This method simply discards an alternative if it has a criterion value below a threshold. To obtain the final score a Pareto optimal strategy can be used (i.e., selects dominating alternatives - better or equal in each criteria- and from this set trade-off analysis can be performed).
- Step 5. Filtering uncertainty - Data should be filtered to consider both lack of accuracy and/or lack of confidence on the collected input data. A filtering function is presented that can handle this type of intrinsic uncertainty in input data. This process corresponds to including some degree of uncertainty (either given from experts or from statistical processes) when there are doubts about the correctness of collected data.
- Step 6. Weighting criteria- we should also consider the assignment of relative importance to criteria (weights). This process should elicit criterias importance, either per iteration or for a set of iterations. For this step we suggest weighting functions [14,15] , because they can handle alternative satisfaction levels per criterion.

In Figure 1.2, we depict the proposed data preparation process with the six steps and their inter-relations.

More details about each step and proposed formulations are discussed using the illustrative example to improve clarity of contributions.

1.2.2. *Phase B. Decision Evaluation Process with Feedback*

In a simple MCDM model this process would be a simple ordering of alternatives regarding the respective rating value, usually denoted ranking of alternatives [3] . However, in a dynamic model, besides using rating values to rank alternatives we also need to include historic information, about preliminary decisions taken along the decision process (iterations). The historical set is refreshed at each iteration, thus ensuring that best classified alternatives, during a certain number of previous iterations, have a better classification in the next iterations. The notion of feedback is introduced to express that at each step (iteration) we build a sub-set of good potential alternatives (historic set) that is re-fed into the system, as input for the next iteration. This feedback continues until a final decision is reached (consensus) or there are no more iterations. In summary, a dynamic decision process with feedback includes three processes for its evaluation of candidate alternatives,

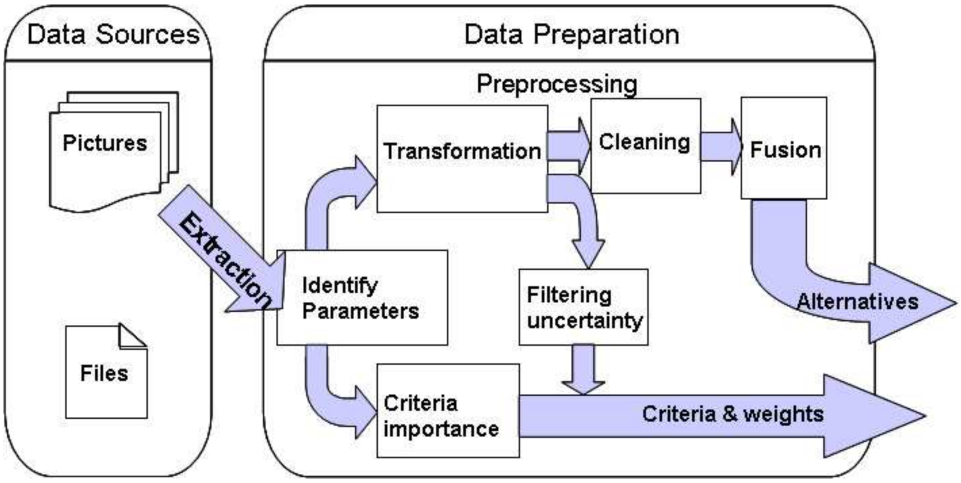

Fig. 1.2. Data preparation process.

as shown in Figure 1.1:

(1) Rating process. During iteration t each alternative is classified, regarding all criteria, using an aggregation method such as the weighted average and weighted with criteria relative importance. There are many aggregation methods that can be used for this purpose [1,7] . The rating process deals with aggregation of all classifications obtained by each alternative in each criterion. During the decision process, we should compute these values for each alternative (per iteration). The proposed formulation for the rating process in our MCDDM is:

$$r_i = \bigoplus(W(ac_{i1}) \otimes ac_{i1}, \ldots, W(ac_{in}) \otimes ac_{in}) \tag{1.2}$$

where $\bigoplus$ is the aggregation operator (in our case Weighted Sum Method); $\otimes$ is the product operator; $W(ac_{ij})$ represents the weighting function for accuracy and confidence (topics detailed in (see Sec. 1.3.2). We propose to compute the weights to express the relative importance of each criterion, using weight generating functions [14,15] :

(2) Evaluation Process. During iteration t, the previous historic set is combined with the rated sites at iteration t (obviously there is only combination if the same site exists in iterations t and t-1). This combination process, henceforth called evaluation process, ensures a continuous refreshment of the historic set for the next iteration (feedback). The dynamic evaluation algorithm for combining historical information ($H(t)$) and current rating values ($R(t)$), is based on the uninorm operator [16] . Our basis is the uninorm operator because it presents full reinforcement behavior[17] and this is what we are looking for in our adaptable and dynamic decision process based on historical data. We refined the uninorm, defining a hybrid

operator [8] because we needed an operator with a compensatory nature outside the T-norms and S-norms[18] interval.

(3) Historic process. The Historic process defines the set k of rated alternatives that are worthwhile considering in the next decision iteration. The set k will record the most relevant information at the end of each iteration. The size of k is context dependent. For instance, if we had 262144 alternatives it would be quite inefficient to pass all data sets from one iteration to another. Hence, for a specific MCDDM we should define an appropriate size for k (best classified alternatives) and then record their respective rating to act as feedback for the next iteration. Noteworthy that we will only mention one historic set because $T = 2$ and the last historic set is $H(T-1)(= H(1))$

There are a wide variety of operators and techniques [1,7,11,16,18] that could be considered to deal with this evaluation process, but, as pointed in previous works [8,12], we propose to use specialized full-reinforcement operators [16,17] because it provides flexibility within the search space. We will not discuss more details about this phase, because our focus is on data preparation in environments where input data changes dynamically, from iteration to iteration (i.e., decision step), and contains some intrinsic imprecision (i.e., we have lack confidence on available data and also have doubts about their accuracy).

1.3. Example in Practice

The example used in this section is a simplification of a real case study [12]. The objective of this multicriteria dynamic decision making case study was to decide which was the best target site, for spacecraft landing on planets. Of particular relevance to an autonomous hazard avoidance system is the ability to select landing sites in real-time as the spacecraft approaches the planet's surface, and to dynamically adjust that choice as better information becomes available along the descent (by performing one or more re-targetings). After several iterations, during the spacecraft final descent, a final decision about the best place to land is achieved [12,19].

In previous works, we have shown good results of using a Fuzzy Multiple Criteria Decision Making approach to solve this problem [8,12,20]. However, we never discussed important aspects related with dynamically changing input data and how the data preparation process ought to be done. This example illustrates interesting contributions for an efficient data preparation process, and also summarizes the decision evaluation process with feedback [8,12] to wrap-up the complete MCDDM approach.

1.3.1. *Background about Landing Site Selection*

Hazard avoidance includes three separate critical functions[12]:

- Hazard mapping that estimates ground features based on imaging sensor data (camera or Lidar), and creates hazard maps;

- Site selection that chooses a suitable landing site based on available hazard maps, mission, propulsion and guidance constraints - The context of this case study;
- A robust guidance to reach the selected target.

The goal of the site selection process is to provide an adequate target-landing site, evaluated with respect to a set of requirements (i.e., criteria):

- The site should be safe in terms of maximum local slope, light level and terrain roughness;
- The site should be reachable with the available fuel;
- The site should be visible from the camera along the descent phase.

In the final phase of descent, starting at around 2.7 km above the surface, the spacecraft is continuously mapping and evaluating the terrain below it, according to a set of criteria (e.g., Figure 1.3 depicts the shadow criterion). It then feeds that information to the site selection component, which will in turn provide the navigation system with the landing coordinates. At all times there is a degree of uncertainty with the evaluations of each site, that decreases (non-monotonically) as the spacecraft approaches the surface.

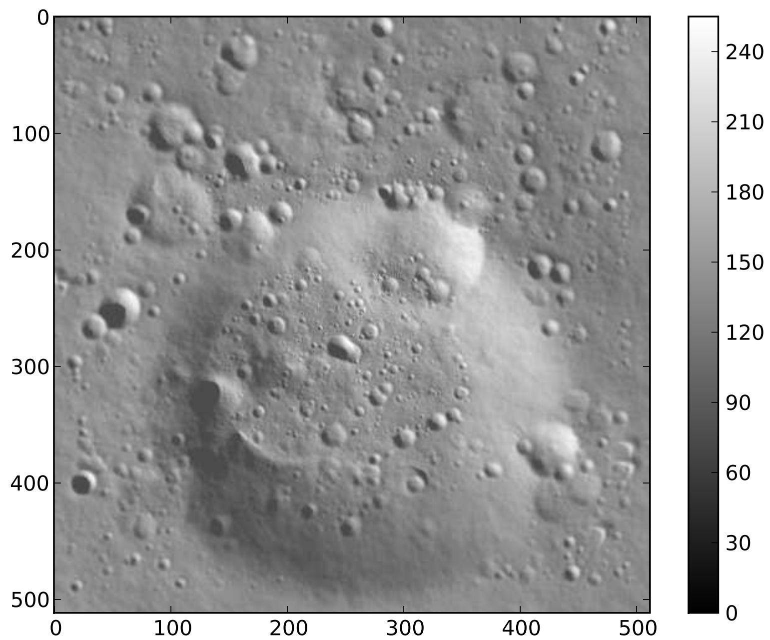

Fig. 1.3. Representation of the Shadow criterion for all sites, in the 8th iteration of the CRATERS dataset

1.3.2. *Phase A. Data Preparation in Dynamic Environments*

Step 1. Identification of parameters To illustrate the approach let us start by identifying the criteria (step 1 of data preparation Sec. 1.2.2). We identified six inputs (hazard maps), each with a matrix structure (512 x 512).

(1) Slope Map - provides the minimum angle to be feared by each pixel, in degrees;
(2) Texture Map - provides the variance value for each pixel;
(3) Shadow Map - provides the values of light in a grey scale (0-255);
(4) Fuel Map - provides the values, in kg, that are necessary to reach the spot corresponding to each pixel;
(5) Reachability Map - provides values between 0 and 1, meaning: 0 if the site is not reachable and 1 if the site is reachable;
(6) Distance Map - provides the distance (in meters) between each pixel to the current target;

An example of a Shadow map can be observed in Figure 1.3 and a texture map is shown in Figure 1.4. All other maps are provided in the same fashion except Distance, which is calculated.

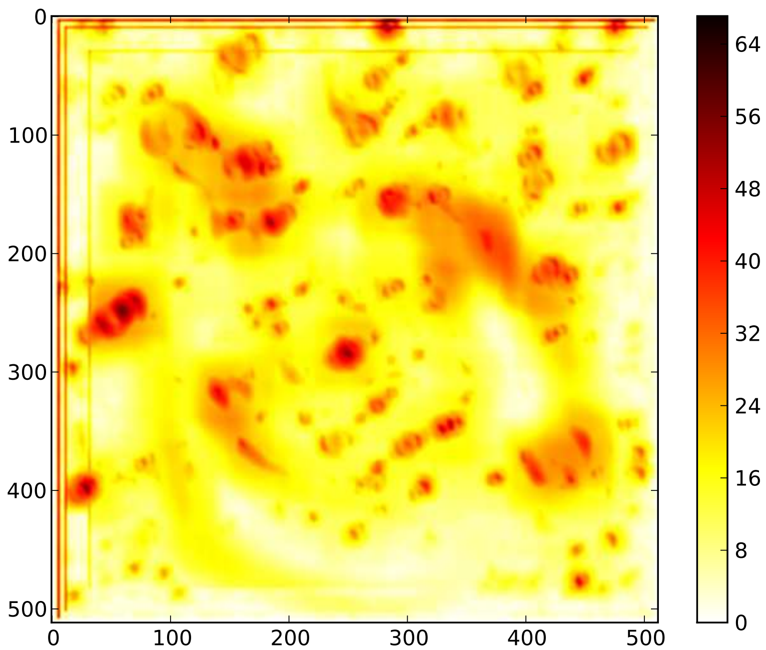

Fig. 1.4. Representation of the Texture criterion for all sites, in the 8th iteration of the CRATERS dataset

Step2. Transformation Now let us deal with Step 2, data transformation. As mentioned, this process tries to answer the question on how to transform input data to be appropriately usable in the decision process. This transformation is two-folded: how to represent the variables and how to normalize the data. An important problem on how to fuzzify variables[2] is to choose the best topology for membership functions because we have to take into account our objective in the decision model. In our case we are trying to choose the best site for landing, i.e., the site, which best fulfils all criteria. Hence, our membership values have to be monotonically increasing towards the best criterion value, i.e., if criterion value is low our membership value is high and vice-versa. Another important aspect to deal with in this step, is to ensure that data is both numerical and comparable. As mentioned, when in presence of both qualitative and quantitative variables, we need to transform and normalize the data to compare them.

For our example we used fuzzy sets[2] to represent and normalize our input variables. To select which was the best topology for membership functions we used available test data to build three well-known functions (from the input data): triangle, trapezoid and gaussian. After, we compare them and select the most representative for usage with our spatio-temporal changing input data.

For example, the membership functions for texture represent "Low Texture Values" because it is what we want to maximize. The variable domain is $[0,255^2]$ but we consider the upper bound to be the maximum value of texture map (see Figure 1.4) for all iterations until the current one. Figure 1.5 shows the membership functions for iteration 23 of a sampling scenario.

Moreover, to define the best morphology for criteria representation, we compared three types of membership functions more commonly used: trapezoidal, triangular and Gaussian. The trapezoidal, triangular and gaussian membership functions were build using the Equations 1.3, 1.4 and 1.5, respectively. Further, consider that x belongs to the interval $[0, upperBound]$.

$$Trapzoidal(x) = \begin{cases} 1 & \text{if } x \leq c \\ \frac{upperBound - x}{upperBound - c} & \text{if } x > c \end{cases} \tag{1.3}$$

where $c = \alpha \times upperBound$, and α defines the range for the function plateau.

$$Triangular(x) = \frac{upperBound - x}{upperBound} \tag{1.4}$$

$$Gaussian(x) = \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{1.5}$$

where $\sigma^2 = \frac{upperBound^2}{2\log(\beta)}$, and β is a parameter that satisfies the following condition: $Gaussian(upperBound) = \beta$

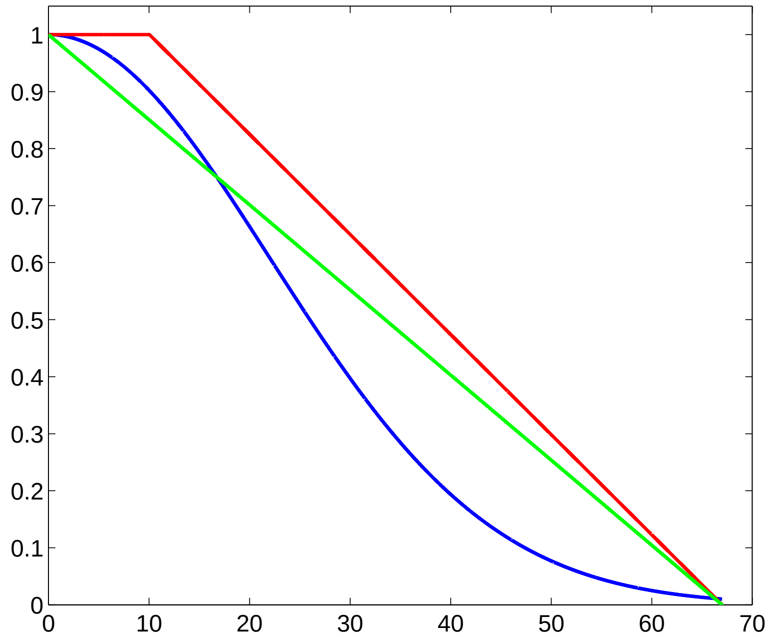

Fig. 1.5. Membership functions for “Low Texture”

Concluding, for representing the Texture map criterion the Gaussian membership function seems the most adequate function because some values, for instance 40, have a too high membership value for triangular and trapezoidal functions when compared with the Gaussian and this can create “bias” in the decision results. Further, Gaussian penalizes high variable values and this is desirable because texture is a problem when values are high.

Step 3. Cleaning In this illustrative case we start by setting to Zero all pixels in the 512x512 input matrix, which have membership value of zero. After, we proceed with other types of “cleaning”, for instance, in the texture example all alternatives that have texture value higher than 68 are discarded. Since for our case study we do not have missing records there is no need to define a strategy to fill the gaps, as other dynamic decision problems might face. Hence, this aspect is not further discussed here.

Step 4. Fusion For fusing the different input parameters to obtain the common set of candidate alternatives for target landing, we used the the conjuntive method[7] with the min operator. With this method we can easily integrate all transformed and cleaned criteria (hazard maps), and obtain the set of 3-D coordinates of possible candidate alternatives. Moreover, this process allows reducing the number of alternatives (search space), which is quite important since there are around 262.000 original alternatives (image pixels corresponding to sites), corresponding to the 512*512 matrices. Conjunctive methods are normally used for discarding unacceptable alternatives[7,21] , hence our choice for the proposed approach.

The procedure for this method is as follows:

(a) Decision maker specifies a minimal acceptable level for each criterion;
(b) For each alternative determine if all criterion values exceed the respective minimal acceptable threshold specified by the decision maker;
(c) If there is one value lower than the minimal acceptable level, the respective alternative is rejected.

Formally:

A_i is acceptable $\Leftrightarrow \forall j : x_{ij} > l_j$ where l_j is the minimal value for the criterion C_j

In Figure 1.6 we depict the final set of alternative pixels that remained after the cleaning and fusion processes took place. It can be observed with these two steps we clearly reduced the number of alternatives to be evaluated.

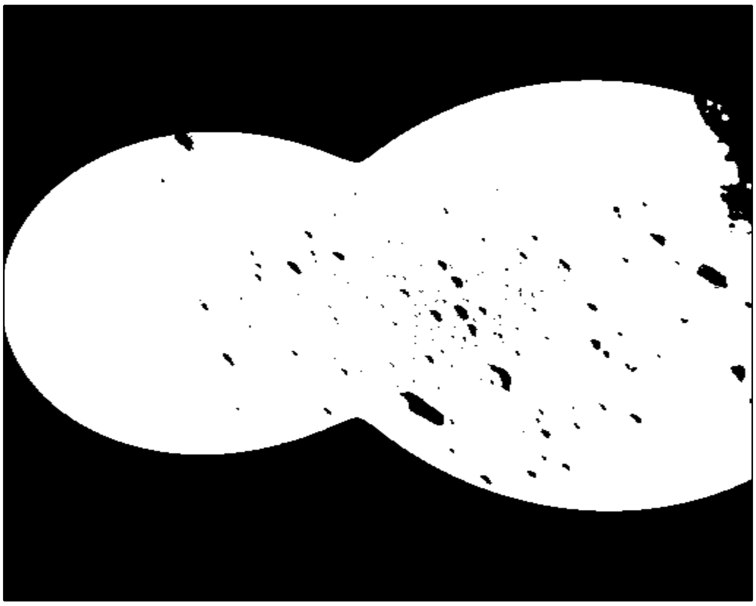

Fig. 1.6. Image of all alternatives, whereas the black areas are those alternatives that have been discarded on the fusion process

Step 5. Filtering Uncertainty Step 5 deals with filtering uncertainty from data. The objective of this task is to deal with intrinsic imprecision/uncertainty found, at each iteration, in the input values. In this case we propose a filtering function that combines metrics to deal with both lack of accuracy and lack of confidence in the collected input values. Formally, the accuracy and confidence parameters (a_{ij} and wc_j, respectively) will be taken into account in the decision model using the following expression:

$$Filter(x_{ij}) = wc_j \times (1 - \max_{x \in [a,b]} \{|\mu(x) - \mu(x_{ij})|\}) \times \mu(x_{ij}) \tag{1.6}$$

where wc_j is the confidence associated to criterion j; x_{ij} is the value of j^{th} criterion for site i; μ is a membership degree in a fuzzy set; and $[a,b]$ is defined as follows:

$$a = \begin{cases} \min(D) & \text{if } x_{ij} - a_{ij} \leq \min(D) \\ x_{ij} - a_{ij} & \text{if } x_{ij} - a_{ij} > \min(D) \end{cases}$$
$$b = \begin{cases} x_{ij} + a_{ij} & \text{if } x_{ij} + a_{ij} \leq \max(D) \\ \max(D) & \text{if } x_{ij} + a_{ij} > \max(D) \end{cases} \tag{1.7}$$

where a_{ij} is the accuracy associated to criterion j for site i; and D is the variable domain.

The accuracy is given as percentage of criterion value and confidence belongs to the unit interval. For example, an accuracy of 90% for slope means that each slope value belongs to the interval $[a,b]$ where $a_{ij} = 0.9 \times x_{ij}$. On the other hand, a 0.9 confidence value means that we have a confidence on the $[a,b]$ interval of 0.9.

We can observe the result of using the $Filter$ function in Figure 1.7. The membership function representing the fuzzy set "Low Slope" is shown with the circle shaped markers, and shown with the triangle shaped markers is the resulting function after using the $Filter$. For each x_{ij} we therefore obtain a filtered value that we call accuracy&confidence membership value, and represent by $ac_{ij} = Filter(x_{ij})$. For instance, in Figure 1.7 we can observe that for a texture value of 20 we have a membership value for "Low slope" of 0.52 and a filtered (with accuracy&confidence) membership value of 0.34. In the example, an accuracy value of $a_{ij} = 5$ is used, for all sites i. That means that a value read as 20 may actually come from anywhere in the interval $[20-5, 20+5]$. The confidence value $wc_j = 0.80$ indicates in general how confident one is with the quality of the process that generates the input values. In this case, the decision maker is indicating there will be room for a 20% improvement in the determination of x_{ij}. Though the criterion may always have a degree of inaccuracy associated with values read, the process whereby they are obtained may be made more reliable.

As it can be observed, this formulation enables dealing with imprecision in the collected input hazard maps. Moreover, it allows dealing with two types of imprecision: lack of accuracy in the data and lack of confidence in the data. We believe this is a simple and easy way to tackle the problematic of how to handle imprecision in data in dynamic decision models. However, in our approach to deal with accuracy we assumed that our decision maker has a pessimistic attitude towards the problem. Hence, Equation (1.6) can be seen as a particular case of a more general approach described by Eq. (1.8).

$$Filter_{\alpha}(x_{ij}) = wc_j \times (1 - \alpha \max_{x \in [a,b]} \{|\mu(x) - \mu(x_{ij})|\}) \times \mu(x_{ij}) \tag{1.8}$$

where $\alpha \in [0,1]$ is a parameter that reflects the attitude of the decision maker (α value

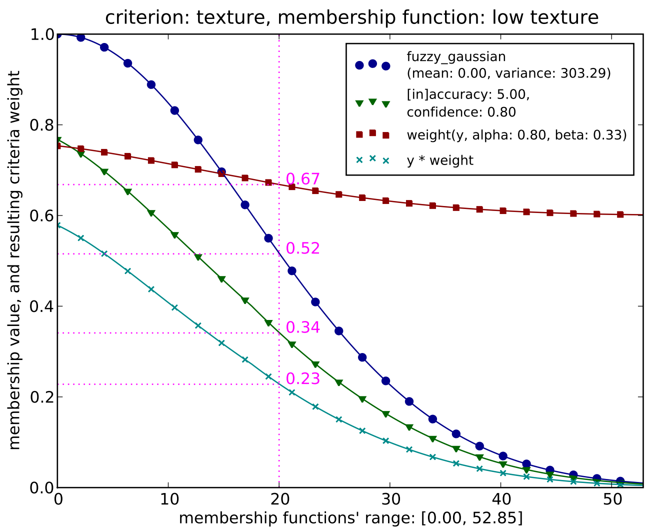

Fig. 1.7. Membership function representing the set of values with *low texture*, before and after applying the accuracy and confidence factors. Given the final membership value, the linear weight generating function of Equation 1.9 is applied, and the criterion's final contribution to quality calculated. The different stages through which an alternative with an initial evaluation of 20 on the criterion passes is also shown.

close to one represent a pessimistic attitude, close to zero an optimistic attitude). All other variables and parameters have the same meaning as in Eq. (1.6).

Step 6. Weighting Criteria This step is the final one for completing the data preparation in MCDDM. It deals with defining the relative importance for each criterion. In dynamic processes it is rather important to consider the satisfaction level, of each alternative for each criterion, to ensure that a poorly satisfied criterion is not "emphasized" by a pre-determined high importance. Hence, we propose to compute the weights to express the relative importance of each criterion, using the following linear weight generating function[14,15] :

$$L_j(ac_{ij}) = \alpha_j \frac{1+\beta_j ac_{ij}}{1+\beta_j} \tag{1.9}$$

where $\alpha_j, \beta_j \in [0,1]$ and ac_{ij} is the accuracy and confidence membership value of j^{th} criterion for site i. Note that ac_{ij} corresponds to the filtered value of x_{ij} from Eq. (1.6).

In Figure 1.7, the line with the square shaped markers depicts the behavior of the weight generating function for the texture criterion.

The logic of these weighting functions (see Eq. (1.10)) is that the satisfaction value of a criterion should influence its assigned relative importance. For example, if we are buying a car and the price is a "very important" criterion, if the car is quite expensive the final decision result should be less than the simple multiplication of weight and satisfaction value.

$$W(\mathbf{ac}) = \frac{L_j(ac_{ij})}{\sum\limits_{k=1}^{m} L_k(ac_{ik})} \tag{1.10}$$

where $\mathbf{ac} = (ac_{i1}, \ldots, ac_{im})$; and L_j is a linear weight generating function presented in Eq. (1.9).

For our case study we considered that the relative importance of criteria has different morphologies for each criterion, depending on the altitude of the spacecraft in relation to the surface. The definition of these weighting functions morphologies is given by the parameters α and β. The α parameter provides the semantics for the weighting functions as follows:

(a) Very Important (VI =1);
(b) Important (I=0.8);
(c) Average importance (A=0.6);
(d) Low importance (L=0.4);
(e) Very Low importance (VL=0.2).

The β parameter provides the slope for the weighting functions, which will depend on the criterion at hand, with the logic that higher values of β means higher slope. In this work this parameter has the following values:

(a) High (H =1);
(b) Medium (M=0.6667);
(c) Low (L=0.3333);
(d) Null (N=0);

For example, for high altitudes ($> 1000m$; $< 2000m$; 250 historic set size), we depict the proposed values for α and β parameters for this phase, and their respective plots (Figure 1.8).

Table 1.1. α and β parameters

	Fuel	Reach	Slope	Dist	Shad	Text	ScIn
α	1	1	1	0.6	0.4	0.8	0.2
β	0.25	0.667	0.111	0.333	0	0.333	0

The rational above is expressed by the α parameter, while the β parameter provides more or less penalties for lower satisfaction values in the criteria, as depicted in the figures for each phase. For example, the rationale for "very important" is that although fuel, reachability and slope are all "very important" in this phase, a lower satisfaction for criteria should be much more penalized in the case of reachability, a little more for fuel and less

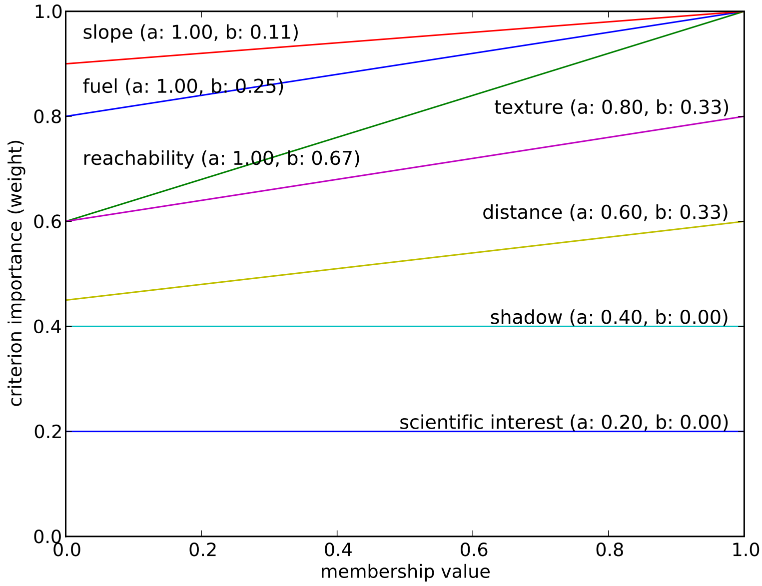

Fig. 1.8. Weighting functions

for slope. In Figure 1.7 we see the weight generating function for the texture criterion. This is an Important criterion ($\alpha = 0.8$) with Low penalty ($\beta = 0.33$) for lower satisfaction values. In the example shown, when $x_{ij} = 20$, we have a somewaht low membership value of $ac_{ij} = 0.34$ to which a weight of $L_j(ac_{ij}) = 0.67$ is attached, down from the criterion's base importance of 0.8. A simplification of the weighting functions could be to define "a priori" a slope for each importance, i.e., avoids having to define different β parameters for each weighting functions. This option simplifies the weight assignment process but it is less accurate. An important point to highlight is that the parameter values proposed for the phases (or iterations) can be tuned for any other dynamic decision applications. More details about this weighting process in the case study can be seen in Ref. 22.

1.3.3. *Phase B. Decision Evaluation Process with Feedback*

Since our scope is dynamically changing input data, in this section we just present a summary of required activities to solve the illustrative example, having in mind the evaluation alternatives when feedback is involved (phase B in Sec. 1.2.2). Details about contributions for this topic can be seen in our previous works[8,12,20] .

The three steps for the evaluation process in our example are:

(i) Rating process - This process refers to the classification of each alternative regarding all criteria, weighted with their relative importance. There are many methods that can be used for this aggregation but a simple and sound method is the Weighted Sum rating method[1] and this is the one we used in our example.

(ii) Dynamic evaluation process - This process refers to the aggregation of historical information with the alternatives rating and then their respective ordering to obtain a set of ranked alternatives. This process is performed at each iteration of the decision process, by combining R(T) with H(T-1) with a hybrid uninorm operator.[8] When there are no more iterations the decision process stops and the best alternative is the one with the highest value.

(iii) Historical process - This process determines a subset of good alternatives, from the evaluated alternatives, to be considered as feedback information from iteration to iteration. Since this process takes place after evaluation, it means the historical information for a specific site "remembers" its past behavior plus the actual rating. We considered different sizes for (K) depending on different phases of the decision process. The K size changed 3 times in our example because it depended on the spacecraft altitude (distance from surface). This set K constitutes the feedback information that is made available for the next iteration.

1.4. Further Research Directions

We presented a general architecture for dynamic multiple criteria decision problems , divided into two main phases: data preparation and dynamic evaluation with feedback. Our scope was spatial-temporal multiple criteria decision problems, requiring feedback from iteration to iteration.

The focus of the work was on the first phase, data preparation, for which we proposed six steps, each with several contributions on how to perform them. We specifically proposed new contributions for dealing with uncertainty in dynamically changing input data and also for dealing with dynamic changes in criteria importance. We believe, these are interesting contributions to improve decision support approaches for MCDDM problems.

Finally, dealing with uncertainty in dynamically changing input data was a challenge and future contributions to improve the evaluation process, both at the level of better operators or new search methodologies to reduce the decision process are foreseeable. In addition, comparison studies between different methods and technologies may also prove to result in improved performance and accuracy for the selection of alternatives in presence of multiple criteria in dynamic decision processes.

Potential applications of this work could be on medical diagnose decision support and/ or fault, detection and isolation (FDI) problems, since both fields require several decision steps and feedback information.

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